AVT-354 Multi-Fidelity Methods for Military Vehicle Design

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Technical Evaluation Report

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1.0 INTRODUCTION

Military requirements continue to evolve rapidly. To meet this growth, the time and effort to develop next-generation weapon systems needs to be significantly reduced, while simultaneously reducing the rate at which problems arise late in system development. Meeting these needs requires new capability for quick, accurate and thorough assessment of the design space. AVT recently completed several assessments of this need.

AVT-ET-054 explored the issue of affordable weapons systems and led to the formation of AVT-092, “Qualification by Analysis,” and AVT-093, “Integrated Tools and Processes for Affordable Weapons Systems.” AVT-093 focused on “the integration of tools and processes, not on the description of tools and processes.” AVT-093 also identified needs in multidisciplinary design optimization (MDO) that could be addressed using the integration of tools and processes in a distributed parallel computing environment that would enable feedback of information from detail to preliminary and preliminary to conceptual design. AVT-092 recognized that these capabilities described in AVT-093 are necessary to achieve the objective of rapid design and qualification of new vehicles. Both teams recognized that there is a gap between the current technology and the desired end state of rapidly developing affordable weapons systems and developments in multidisciplinary technologies are key capabilities for closing that gap. More recently, AVT-237 focused on benchmarking the use and benefits of MDO for the development of military systems, and AVT-252 explored optimization of aircraft and ships under uncertainty.

Finally, the AVT-331 team is discussing, developing, and applying methods for accelerating vehicle design by using tools and processes that reflect different levels of fidelity throughout the MDO process and recommend broadening the discussion in our community via cooperative events. To this aim, AVT-331 co-chairs and members organized a special session on Multi-Fidelity Methods for Vehicle Applications at the Multidisciplinary Analysis and Optimization Conference within AIAA Aviation Forum and Exposition in 2020 and, along the same line, proposed this RWS, to be held in 2022. A cooperative event discussing interim findings of AVT-331 will help establish a broader dialogue regarding the technical and military relevance of AVT-331 findings and will help identify potential follow-on efforts to heighten the benefits of the target methodologies.

2.0 THEME

The above mentioned previous AVT efforts have consistently shown that there is design benefit to coupling more engineering disciplines at higher levels of fidelity earlier in the development process. But there is no mathematical framework to determine which disciplines, which level of coupling, or which level of fidelity is required to capture the physics most critical to a particular system’s design, or how to make the best possible design decision with constrained computing resources.

A better understanding of frameworks, architectures, and methodologies for the adaptive selection of disciplines, fidelities, and coupling for modelling, simulation, and finally optimization of military systems is anticipated to provide the following benefits:

1) Decrease late discovery of system defects due to misunderstood physics through improved modeling of physics in design – decrease military vehicle development time and cost.
2) Increase the available design space with more disciplines and by potentially leveraging physical interactions – increase system performance and generate new capabilities.
3) Decrease time and resources required to execute the MDO process through goal-oriented approaches – accurate information provided faster to NATO decision makers.

3.0 PURPOSE AND SCOPE

The objective of AVT-354 is to facilitate the identification and possibly the extension of the current state of the art associated with frameworks, architectures, and methodologies for the adaptive selection of different sources of information from data/models for design of military vehicles.

The RWS will cover a broad range of topics in the multidisciplinary design optimization of military vehicles and emphasize how the incorporation of multi-fidelity methods in the design process can accelerate design procedures and enable more accurate physical analysis.

Experts are invited to present methods and applications across the military vehicle space and address benchmark/mini-problems for assessment of methods and discussion on future needs and capabilities.

The Workshop will cover a broad range of topics in the multidisciplinary design optimization of military vehicles, with specific emphasis on how the incorporation of multi-fidelity methods in the design process can accelerate design procedures and enable more accurate physical analysis. Papers and posters are invited that have relevance to the following questions (such questions will be used to focus workshop discussion):

1) What are the available methods to synergistically fuse information sources of different fidelity to accelerate multidisciplinary design optimization and how do these methods scale with the size of the design space and the addition of different disciplines?
2) How can fidelity decisions be based on system-level objectives and constrained by available computational resources?
3) What is the best way to blend multiple sources of test and computational data, and what is the impact of noise in any information source?
4) What are the outcomes of existing multi-fidelity benchmarks and where do these benchmarks need to be improved or extended?
5) How can multi-fidelity modeling be used to address multiple sources of error, enable efficient multi-fidelity uncertainty quantification and robust optimization of complex systems?
6) What are the connections between multi-fidelity modeling and machine learning methods?
7) Where does future NATO work need to be carried out to resolve remaining challenges in this topic area?

The focus of the workshop will be on design applications for sea, air and space platforms, but contributions regarding generic tools and techniques and best practice from other domains will be welcome. Applications for complex engineering problems such as material design, propulsion systems, autonomous systems and energy systems will be of interest.

4.0 EVALUATION

Three keynote presentations and 14 paper presentations took place during the meeting.

Summary of Presented Contents

The following pages present short summaries of each of the keynote, paper and poster contents.

***Keynote 1* – MSTC Engineering – A Computational Framework for Multi-Fidelity MADO**

Author: Raymond Kolonay, AFRL, United States

Dr. Ray Kolonay's presentation focused on the activities at the Multidisciplinary Science and Technology Center in the area of multi-fidelity methods. MTSC Engineering is a physics based computational analysis and design environment based on JAVA and it consists of a distributed collaborative multidisciplinary, multi-fidelity and multi-scale analysis and optimization framework with uncertainty quantification. The MADO N3 architecture was explained and a description of the building blocks encompassed the development, assessment and validation. Target applications included supersonic aircraft.

***Keynote 2* – Reduced Order Modelling and Parameter Space Reduction in Engineering: Fluid and Structural Mechanics Problems**

Author: Gianluigi Rozza, SISSA, Italy

Dr. Gianluigi Rozza's presentation focused on parameter space reduction and data-driven model order reduction. He presented some benchmark academic applications and also the application to naval engineering. This was a very insightful presentation that spanned linear active subspaces, linear parameter space reduction, kernel based active subspace extension. Examples included a CFD application with an internal flow. He also addressed the constrained global optimization problem of a real-valued continuous function in the context of genetic algorithms. An interest application was a naval engineering optimization with advanced morphing.

***Keynote 3* – Decision Making Support through Optimisation**

Author: Nadir Ince, GE, United Kingdom

Dr. Nadir Ince's presentation was more focused on the industrial perspective. A decision making process was discussed and its relation to digital transformation. Automation in optimization was also discussed. Other topics presented included uncertainty quantification and model verification. Physics-based Neural Networks may improve the reliability of models. Overall, an interesting industrial perspective.

***Paper 1* – Comparison of Multi-Fidelity Approaches for Military Vehicle Design**

Authors: Philip Beran, Dean Bryson, Andrew Thelen, Matteo Diez, Andrea Serani and Laura Mainini

The paper describes the context established for the comparison of multi-fidelity (MF) approaches by the AVT-331 group. A categorization of benchmark problems by 3 levels of increasing complexity (from analytical to multidisciplinary optimization problems) based on qualitative measures of mathematical nature of the quantities of interest, accuracy of the numerical tools used, computational costs and similarity to real world problems. Furthermore, the paper describes the categorized benchmarks problems used by the AVT-331 group, establishing important criteria for MF approaches comparison, as p.e. computational budget and number of design variables used. Furthermore, the MF methods under comparison are categorized as well as the assessment criteria.

Paper 2 **– Analytical Benchmark Problems for Multifidelity Optimization Methods**

Authors: Laura Mainini, Francesco Di Fiore Andrea Serani, Simone Ficini, Riccardo Pellegrini, Matteo Diez, Markus P. Rumpfkeil, Edmondo Minisci, Domenico Quagliarella, Hayriye Pehlivan, Sihmehmet Yildiz, Melike Nikbay, Dean Bryson and Phil Beran

This paper follows on paper#1 as it describes first level of complexity benchmarks used by the AVT-331 group for multi-fidelity (MF) methods assessment. It provides a compilation of useful information on existing analytical functions and their MF extensions, including the optimum location, fidelity costs considered and computational budgets, establishing this way a common ground for the assessment of the MF methods. Additionally, metrics associated to the real function approximation (goal insensitive) and optimum approximation (goal sensitive) are proposed, as well as an aggregated metric to be used in MF methods assessment.

Paper 3 **– Reproducible Industrial Multifidelity Optimization Benchmark Problems for Air, Space, and Sea Vehicles**

Authors: Domenico Quagliarella, Andrew Thelen, Daniel Clark, Dean Bryson, Phil Beran, Sihmehmet Yildiz, Melike Nikbay, Laura Mainini, Francesco Di Fiore, Edmondo Minisci, Penelope Leyland, Andrea Serani, Simone Ficini and Matteo Diez.

The paper describes the information, software and results of the complexity Level 2 benchmark optimization problems proposed by the AVT-331 group required for reproduction of the results and to be used for multi-fidelity methods assessment. The problems include air, space and sea vehicles benchmarks.

Paper 4 **– Recent Improvements in Spatial Regression of Climate Data**

Authors: Jouke H.S. de Baar and Irene Garcia-Marti

The paper describes an interesting real-world application of the multi-fidelity (MF) Kriging surrogate model to make spatial regression of climate data in the Netherlands, resorting to the inclusion of high resolution, low fidelity data from covariates together with high fidelity data in the model. A treatment of noise in measurements is proposed and its importance demonstrated in a context of spatial regression of MF data.

Paper 5 **– Resistance and Seakeeping Optimization of a Naval Destroyer by Multi-Fidelity Methods**

Authors: Andrea Serani, Riccardo Broglia, Matteo Diez, Gregory Grigoropoulos, C. Bakirtzogou, NTUA, Greece, Omer Goren, D.B. Danisman, Hayriye Pehlivan Solak, Sihmehmet Yildiz, Melike Nikbay, Thomas Scholcz and Joy Klinkenberg

The article addresses the optimization of a Naval destroyer considering three different objectives. With a high number of evaluation tools and different multi-fidelity methodologies it provides a broad and insightful benchmark for the research area. After an introduction of the problems and the parametrization for the optimization, the different numerical solvers are summarized, containing a variety of fidelities. Subsequently the different multi-fidelity methods are introduced and the frameworks for the optimizations described. A result comparison and analysis is then scrutinized for the frameworks. Noticing the strong collaboration between a high number of researchers and the extensive content of the paper, it shows the strength of the specialists' meetings of combining expertise and finding synergies in a multidisciplinary environment. The paper contains multiple aspects on methodologies, approaches, tools of different fidelities and assessments

Paper 6 **– Efficient Hull-Form Optimisation Using Multi-Fidelity Techniques**

Authors: Thomas Scholcz and Joy Klinkenberg

The paper details the formulation of a Multi-fidelity Kriging method and an associated adaptive training method based on an augmented Expected Improvement formulation. A thorough discussion of the methods results obtained for analytical and numerical benchmark problems follows by assessing design space, objective function and global error metrics. The different phases of the Kriging prediction along the optimization are detailed and the behavior of the error metrics allow for an understanding of the differences between single fidelity and multi-fidelity methods. The accuracy of noise prediction is also addressed.

Paper 7 **– Compositional Kernels to Facilitate Multi-Fidelity Design Analysis and Optimization: Applications for Early-Stage Ship Design**

Authors: N.D. Charisi, A.A Kana and J.J. Hopman

The paper proposes and tests a methodology for multi -fidelity design analysis based on compositional kernels. The assessment of the methodologies used benchmark analytical function results and a simplified design problem. The paper results demonstrate the applicability of the method and its advantages. Studies on the method's performance under varying number of HF data and problem dimensionality confirm its superiority relative to a squared exponential kernel Kriging model in terms of fitness to the function to be represented.

Paper 8 **– Adaptive Multi-Fidelity Metamodelling for High-Quality Shape Optimization**

Authors: Riccardo Pellegrini, Andrea Serani, Matteo Diez, Jeroen Wackers, Hayriye Pehlivan Solak and Michel Visonneau

The paper describes a multi-fidelity model formulation based on stochastic Radial Basis Functions and proposes an active learning method to be used in high quality optimization. The active learning method is intended to distribute the available sampling budget among the fidelity levels efficiently by including a mean square error term to prevent addition of noise sampling points to already accurate fidelity levels. The method is applied to an analytical test function, an airfoil and a hull shape optimization problems showing that the active learning method can significantly improve the computational cost in optimization.

Paper 9 **– Comparison of Multi-Fidelity Optimization Methods Using an Aero-Structural Benchmark Problem**

Authors: Dean Bryson, Philip Beran, Andrew Thelen, Markus Rumpfkeil, Melike Nikbay, Enes Cakmak and Sihmehmet Yildiz

The article assesses two elaborate, aeroelastic, benchmark optimization problems with multi- and single-fidelity methodologies. The results are analysed regarding their performance on found optimum and convergence speed. All methods could improve the objective reasonably, with little deviations in the results. However, the results were inconclusive if one of the other approach is superior at this point. Yet, a first step is taken towards a comparative study to assess different methods, which can provide a starting point for further research.

Paper 10 **– Multi-Fidelity Shape and Mission Optimization Including Transient Thermal Constraints**

Authors: Christopher A. Lupp, Daniel L. Clark Jr., Christopher T. Aksland and Andrew G. Alleyne

The paper describes an optimization procedure for a HALE aircraft considering aerodynamics, propulsion and Power and Thermal Management Systems. The optimization includes elements of multi-fidelity in the sense that high-fidelity data is used with lower fidelity analytical models for aerodynamic coefficients calculation, which deviates from the majority of the multi-fidelity applications in AVT-354.

Paper 11 **– Development of a Multi-Fidelity Optimization Framework in MDO with Application to Aeroelasticity and Aeroacoustics**

Authors: J. Lobo do Vale, M. Sohst, F. Afonso and F. Lau

This paper focuses in applying and attempting to compare the Multi-fidelity Kriging and Deep Neural Network-Multi-fidelity Bayesian Optimization approaches to MF MDO in test functions, aeroacoustic and aeroelastic problems. The multi-fidelity Kriging outperforms the single fidelity in the initial optimization phases in every case for the test functions and the aeroacoustic problems. For the aeroelastic problem only an assessment of prediction accuracy was performed using the LF data, showing that predictions deviate more than the estimated uncertainty in a significant number of times, evidencing difficulties in the use of the surrogates for feasibility prediction. Using the DNN-MFBO approach revealed that the implementation was probably flawed and that the sensitivity of the MF model to the hyperparameters initialization is an additional difficulty.

Paper 12 **– Modeling Hypersonic Vehicle Performance and Operations using a Multi-Fidelity Reduced Order Modeling Approach**

Authors: Kenneth Decker and Dimitri Mavris

This paper compares several Reduced Order Modeling (ROM) techniques, including linear and non-linear ones, applied to the prediction of aerodynamic coefficients and optimization of trajectories of a hypersonic vehicle. Furthermore, Manifold Alignment and Proscrutes analysis are used to capture low and high fidelity data in a mapping of the high dimensional to the low dimensional space that encapsulates all available information. The hypersonic vehicle problem is used for comparing single-fidelity ROM as well as Multifidelity ROM.

Paper 13 **– Conceptual Level Sizing, Evaluation and Design Space Exploration Tool Utilizing a Surrogate Model**

Authors: Hasan Ibaçoğlu, Abdullah Enes Coşkun and Tolga Kayabaşi

The paper describes the process of helicopter design on a high level. The design aspects for important subcomponents and design decisions are explained in detail. Surrogate models are represented by response surface models and claimed to be used for the design, however, appear only once in the conclusion of the text. This aspect should be extended if mentioned with such prominence in the title. The multi-fidelity content lacks depth, since only shortly mentioned in the beginning of the article.

Paper 14 **– Design Optimization based on Metamodels combining Multi-fidelity simulations**

Authors: Alberto Clarich, Luca Battaglia, Lucia Parussini, Haysam Telib and Angela Scardigli

The paper describes the incremental application of methods to increase the accuracy of metamodels generated from a limited number of high fidelity (HF) data samples and additional low fidelity (LF) data samples. The methods include a Dataset Reducer that selects the best HF Design of Experiments (DoE) based on the quality of a metamodel trained and validated using LF data, followed by using the generated HF and LF data together with a co-kriging formulation (in the case of scalar fields) or a ROM Multifidelity (MF) formulation (in the case of vector fields) to further improve the quality of the resulting metamodel. Application to an aeronautical test case showed that the quality of the metamodels is significantly increased by using the Dataset Reducer method and further improved by including MF methods for the scalar field predictions. Application to an aerodynamics case with vector fields and MF ROM showed a drastic decrease in the prediction computational time when compared to HF and LF models with acceptable accuracy.

Meeting Objectives and Achievements

The following pages relate the meeting objectives with the presented research and reports the main conclusions for each objective. In particular, these objectives assume the form of summarizing answers to the questions stated in section 3.0. These answers stem from the understanding of the evaluator given the presented contents and discussions during the RWS.

1) **What are the available methods to synergistically fuse information sources of different fidelity to accelerate multidisciplinary design optimization and how do these methods scale with the size of the design space and the addition of different disciplines?**

   The majority of exploratory methods employed in the presented papers use Gaussian processes as basis to integrate different fidelity information into a predictive tool. Multifidelity approaches based on different kernels (Gaussian exponential, Compositional kernels, Partial Least Squares) were explored and Multifidelity Kriging was the most popular choice. Other multifidelity approaches were based in Radial Basis Functions, Reduced Order Models and Deep Neural Networks.

   Scaling continues to be a challenge, although some proposals for dimensionality reduction (Parametric Model Embedding, Reduction to latent space), have proven to be useful.

   Although multidisciplinary optimization problems were used as benchmarks in some cases, the effects of problem multidisciplinarity in the computational cost of MF MDO were not addressed in a significant extent, perhaps because the MF approaches are agnostic to what is considered high fidelity data (p.e. higher discretization in CFD mesh vs inclusion of aeroelastic deformation in the analysis).

   Main Conclusions:

   a) MF in general accelerates the efficient exploration of the design space and this occurs for almost all of the methods presented in the RWS, but this acceleration is in fact dependant on the cost ratio between fidelities used in the MDO procedure. It seems clear that augmenting the surrogate models with low fidelity data improves their accuracy and reduces uncertainty in predictions.

   b) Dimensionality is a problem, although there are methodologies that soften this problem. In practice, it is desired that non-influential design variables are excluded from the MDO problem or that there is a reduction of two or more variables to one parameter describing their influence in the predicted parameters, which is accomplished by p.e. Reduced Order Models and Parametric Model Embedding.

   c) The Multidisciplinarity of the problem has not been specifically addressed regarding its impact on MF methods.

2) **How can fidelity decisions be based on system-level objectives and constrained by available computational resources?**

   A number of approaches include considerations of fidelity relative computational costs and amount of information gain on the optimum in their search functions, although these cost metrics do not have a particular reasoning behind them and stem from the context of the research groups presenting in this RWS. The question of availability of resources within project time, for instance, does not seem to have been considered.

   Main Conclusions:

   a) The computational cost vs information gain has been addressed and is inherent to some of the methodologies presented.

   b) The definition of the information gain and computational cost metrics is still lacking standardization and objectivity. This is not an easy quantity to define but it does have a strong impact in the way the MDO search path.

3) What is the best way to blend multiple sources of test and computational data, and what is the impact of noise in any information source?

From the presented results one can say that some methods are more likely to speed up the MDO process than others, but there is no certainty that there is a particular method that is significantly better than the others. In fact, it has been shown that MF approaches are not always better than single fidelity approaches in terms of computational costs.

Several approaches to introduce noise in the MF methods have been presented in this RWS and all seem sound and producing more accurate results compared to (assumed) noise free information.

Main Conclusions:

a) The efforts of the AVT-331 team are currently producing knowledge about several MF approaches and further analysis of results is required to rank methodologies with objective criteria. The definition and analysis of several level benchmarks has been a great contribution to this goal.

b) Noise impacts negatively the contribution of LF data to the MF surrogate model if not included in the MF method formulation.

4) What are the outcomes of existing multi-fidelity benchmarks and where do these benchmarks need to be improved or extended?

The AVT-331 team produced several level benchmark problems providing a strong base for MF methods evaluation and assessment. The range of problems is very adequate for the current state of the art of MF MDO research.

Expansion of the benchmark problems could be made towards increasing multidisciplinarity (and consequently dimensionality also) of the MDO problems in order to motivate the development of methodologies to tackle an eventual complexity increase in both the problem and the MF methods used.

Main Conclusions:

a) The benchmark efforts so far have still to help produce a ranking of MF methodologies. It seems too early to introduce extra elements of complexity in the benchmark problems before one can confidently use them for MF methods classification. Nevertheless, moving towards higher multidisciplinarity is suggested for further benchmark developments.

5) How can multi-fidelity modeling be used to address multiple sources of error, enable efficient multi-fidelity uncertainty quantification and robust optimization of complex systems?

The presented approaches deal with error by introducing noise distributions, typically Normal, in the formulation of the MF models, allowing the determination of the posterior distribution of the quantities of interest. This posterior distribution has an uncertainty associated with it which, depending on the model assumption, can be quantified analytically or through some type of approximative technique. Using that uncertainty when modelling constraints one should be able to establish a robust optimization problem that makes use of it.

Main Conclusions:

a) The error and uncertainty quantification have been addressed rather successfully within the RWS related works, but there was not much research in its application to robust optimization.

6) **What are the connections between multi-fidelity modeling and machine learning methods?**

There is a common trait between the kernel based approaches and Neural Networks which is the necessity to fit the models based on the optimization of hyperparameters. When the NN hyperparameters optimization (training) is based on the same maximization of likelihood assumptions than kernel based approaches, one has high similarity between the two approaches.

Some concepts of information theory can be applied to MDO and MF methodologies, namely in the search function definition. One possible benefit of using NN is the availability of mature computational tools to build and train those NN and the freedom in NN architecture that might contribute to higher MF accuracy. On the other hand, this freedom creates additional challenges in the NN architecture definition.

Main Conclusions:

a) When framed in the MDO context, the surrogate models used more traditionally in MF MDO (Gaussian processes) can be replaced by NN and make use of the training tools available in machine learning methods. Furthermore, knowledge from information theory can be used in the formulation of search and training functions.

b) The NN based MF approaches should be further explored and compared to the presented MF methods in this RWS.

7) **Where does future NATO work need to be carried out to resolve remaining challenges in this topic area?**

a) In the immediate future, finalizing the efforts by the AVT-331 team and conclude about the different MF methods, possibly narrowing down to the most promising methods and further expand their application to all the benchmarks.

b) Increase the multidisciplinarity of the benchmarks and test the MF methodologies.

c) Exploration of NN based MF methods, understanding their pros and cons, applicability and comparing them with the previously developed methods is strongly suggested, given the already existent literature published by computational sciences groups on the subject.

5.0 CONCLUSIONS

- The NATO-AVT-354 workshop presented the efforts of the AVT-331 team in establishing different level benchmarks for the assessment of MF methods. Additionally, the AVT-331 team and other groups have proposed and explored different MF methods ranging from kernel based approaches to reduced order models and Neural Networks.
- Definitive conclusions about the ranking of the proposed MF approaches in terms of global accuracy and objective driven metrics are still to be drawn but underway.
- There is still no standard for attributing fidelity costs to different fidelities. This may not have an impact on the MF methods assessment, but may have in the choice of using MF vs SF. Current state of the art blends the expected information gain on the optimum with the fidelity cost in the search function to determine the next fidelity and evaluation point.
- Noise must be considered in MF modelling in order to achieve a suitable accuracy for both global and local model representativity.
- Both Gaussian processes and NN based approaches can use similar training and search function formulations, meaning that practices and methods used in machine learning can be used in the implementation of MF MDO approaches.

- Overall, although significant progress has been made, there is still a final stretch to make in order to achieve some better establish the MF methods and their efficiency and effectiveness metrics. There are also other less explored avenues that deserve more effort in understanding their applicability in MF MDO, as are the cases of ROMs and NN.

6.0 RECOMMENDATIONS

- Finalize the studies proposed in the AVT-331 group.
- Attempt to standardize fidelity costs metrics.
- Benchmark improvements including increased multidisciplinarity.
- Further exploration of ROMs and NN based MF approaches.

7.0 APPENDIX – ROUND TABLE DISCUSSION

In the morning of Day 3, two round-table discussions took place. The following questions and associated observations summarize the outcomes that emanated from the group discussions.

1) **How can multi-fidelity modelling be used to address uncertainty quantification and robust optimization of complex systems with multiple sources of error?**
   - Robust design needs to be defined more clearly. Selection of control variables that make the design as insensitive as possible to uncontrollable factors.
   - Uncertainty quantification needs to be extended to account for all the phases of UQM. Identification, propagation, quantification, mitigation and control.
   - Uncertainty variable sensitivity analysis at system level can provide a means for fidelity level determination.
   - Need more care in defining what we mean by it. Some of the presentations were using the same tool/physics but they were modifying the grid density.
   - Probability distributions need to account for more than the mean and variance. Tails are important for many disciplines (reliability, fatigue). Need to run a very large number of cases. Surrogates + Monte Carlo techniques or MC approximation techniques such as polynomial chaos expansion, FPI technique.
   - Codes with internal constraints/optimizers might cause issues with surrogate formulations.
   - Design space exploration and conceptual design often have no feasible space unless technologies are infused to “open” the design space.
   - Need to move from scalar responses to field and vector surrogates. x,y,z,t mappings+ operating conditions+ vehicle orientations + OML variations to make it a real design problem.
2) **What are the synergies between multifidelity modelling and machine learning?**
   - LF is typically fast and not accurate, HF is typically slow but more accurate.
   - ML is data driven and has the potential to improve / expand MFM (multi-fidelity methods) and can generate models that are fast and accurate within the adequate parameter range. However, it may be expensive to generate / collect data required for ML.
   - UQ is particular important in this context. Need to discuss / research both statistical uncertainty and model uncertainty.

- ML can potentially improve the MFM process.
- Physics-informed machine learning (PIML) is particularly attractive. Physics can help to guide ML with existing physical principles, and ML can use data to bring models (physics-based) to reality.

3) How could the AVT-331 benchmarks be improved, extended, or replaced?

- Definition of the L1 problems were found on arXiv and were helpful at early stages of development.
- For the L2 & L3, some sort of tutorial would be helpful, or at least some limited documentation of the baseline.
- The tools are mostly open-source, but there are a few licensed packages (e.g., MATLAB, Nastran).
- Software installation is not trivial. Where do users go when they need help? Is there some sort of forum or Q&A?
- There is some excitement that there is a starting point for some multi-fidelity problems, but if installation is hard, people will give up.
- Also, repeatability and robustness can be difficult across machines/platforms.
- Are the L1 problems appropriate for neural nets? The surrogate modelling community focuses on Forrester and others, but we do not see this in the Machine Learning community.
- Relatively small problems presented and discussed—nothing really large scale.
- Nonlinear autoregressive Gaussian processes are able to provide uncertainty quantification and have been proven as a viable option for optimization of large-scale systems. If multi-level methods are available, we can exploit analytical error estimates
- Machine learning has the potential to enhance the information fusion characterizing multi-fidelity methods.
- Maybe to create large scale benchmarks with data from industry.
- Convergence rates for a given computational budget in both data scarcity and big data regimes.
- Accounting for noise in the data is a fundamental aspect for every multi-fidelity method. It should be naturally incorporated during the analysis or mitigated by using filtering methods.

4) What assessment approaches would be useful for AVT-331 to employ to fairly compare multi-fidelity methods?

- It really helps if the surrogate is well-studied and also allows for interpretation (my addition: for example in kriging / rbf the kernel shape, length scale and noise levels allow for interpretation).
- Cross-verification of the final design. The performance of the design found by one participant is also computed by other participants.
- Computational speedup (= MF cost compared to SF cost) could be presented as a function of desired accuracy.
- Also quantify other cost (or lead time) aspects, like learning to use different meshers/solvers, surrogate overhead (hyperparameter estimation, expected improvement optimization). Related question: should we focus on cost or on lead time?

5) How can multiple sources of test and computational data be blended, and what is the impact of noise arising from one or more information sources?

- How to define “fidelity”? Is it only accuracy or a combination of accuracy and reliability of the data (e.g., presence of outliers)?

- It depends on whether it is possible to define the reliability and the accuracy of each source: is it possible to establish a high / low fidelity relation? If this is not possible, then data fusion could be considered.
- Different types of blending:
  - Fusion (weak hierarchy is supposed);
  - Calibration (one source is clearly superior over the other);
  - Filtering.

6) **What are the methodological challenges associated with practical use of multi-fidelity approaches?**
   - The main problem for large scale application is the robustness of the methods. Good regressions should be achieved with minimal user interventions.
   - Having a common set of benchmarks is crucial to compare different multi-fidelity methods and allow the end user to pick the best one for the problem at hand. Uncertainty quantification and the use of risk measures in the context of design optimization should be further explored.
   - The main problem for large scale application is the robustness of the methods. Good regressions should be achieved with minimal user interventions.
   - Risk-based engineering design optimization techniques.

7) **What directions/activities would be useful for NATO to explore as a follow-up to AVT-331 and this workshop?**
   - More about decision making, and how we bring decision making into account.
   - How can MF methods help in the design process / decision making? How much information do you need at each stage of a design? Perhaps use a decision making benchmark? It appears interesting to study multi-fidelity as an aid to decision making. This raises questions beyond the pure functionality of the methods. The value of information, related with the fidelity, could vary during a design process.
   - Reaching out to the AI community could be interesting. A critical issue is to make this community understand the specific questions of design, such as the high cost of data.
   - Lecture Series is the easiest way to convey information/learn is orally. Potential lecture topics: deeper into benchmarks, industrial applications, end with how we close the loop with decision making.

8) **What are the barriers to transition of multi-fidelity approaches to industrial design of military vehicles?**
   - Biggest barrier is the fact that industry is organizationally fragmented at preliminary and detailed design, manufacturing… extremely difficult to apply our MDO formulations. They are viewed as academic!
   - Surrogate methods are a great enabler to remedy the situation. Let the experts/ entities responsible for the various analyses, to carry out their work as usual…expect a design space mapping rather than point solutions. Transfer functions exchanged rather than tools and methods.
   - Unfamiliarity with a lot of the methods described. Engineers are not particularly math savvy. They need automated tools and well defined use cases/tutorials in laymen terms.
   - Recognize that MF methods that had a degree of "intuitiveness" would have a greater likelihood of acceptance by the engineering (rather than research) community.

- Main barrier: lead time and flexibility.

  Possible solution: offline learning of surrogate and online interrogation/exploitation (possibly sitting around the table with the end user and using a GUI).

9) **What didn't you see at this workshop that would have been beneficial?**
   - Additional methods, different applications, uncertainty quantification and robust design, incorporation of experimental data, cross-panel benchmarks, etc. The workshop was very design centric. Include image processing, signal processing and videos of unsteady flow processing.
   - Sharing of surrogate software / scripts.
   - Link between MF and machine learning.
   - Detail of the MF Methods.
   - Performance comparison of different methods.
   - Experiment / test results / validations.

10) **How can fidelity decisions be based on system-level objectives and constrained by available computational resources?**
    - Full system models rely on fast evaluation of subsystem performance. A full system model cannot really benefit from a low-fidelity equivalent. However, for the subsystem models, accurate models that are fast to evaluate are crucial for the decision making. A question is, how the reliability of these subsystem models should evolve as the design progresses. Multi-fidelity models could play a useful role here.